

WORD PROBLEMS: FROM THEORY TO STRATEGIES

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WHY STUDENTS STRUGGLE TO SOLVE WORD PROBLEMS?

- The scenario represented is outside the experience of a student.
 - The vocabulary is unfamiliar.
 - The numbers are larger than the student is able to compute mentally.
 - The student was unable to identify the underlying schema of the problem.
-
- Students who have difficulties in math see every problem as a brand new problem.



WHAT HAVE WE BEEN TEACHING?

- The Problem-Solving Process
 - Understanding the problem,
 - Devising a plan to solve the problem,
 - Implementing the plan, and
 - Reflecting on the problem.

DIFFERENT STRATEGIES TO SOLVE WORD PROBLEMS

- Draw a picture
- Choose an operation
- Make a table
- Act it out
- Work backward
- Guess, Test, Revise
- Work a simpler problem
- Make an organized list
- Find a pattern

NCTM Strategies

Strategy	%
Draw a picture	40%
Choose an operation	20%
Make a table or graph	17%
Act it out	9%
Work backwards	6%
Guess, test, revise	4%
Work a simpler problem	2%
Make an organized list	1%
Find a pattern	1%

STRATEGIES *NOT RECOMMENDED*

BY NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (NCTM)

Specific Strategies Taught to Students for Problem Solving

**Does NOT
work**

Strategy	2nd	3rd	4 th	5th	Multi	Total	%
Identifying key information	5	6	8	11	1	31	36%
Clue words	3	6	6	3	0	18	21%
Eliminate extra information	2	5	2	3	1	13	15%
Reread	4	4	1	0	0	9	10%
Writing own problems	0	0	4	1	0	5	6%
Understand, Plan, Solve, and Evaluate (UPSE)	2	1	2	0	0	5	6%
Reword	1	1	0	1	0	3	3%
Replace names	1	0	1	0	1	3	3%

DRAW A PICTURE

Jenny has \$12 in the bank. She earns \$5 doing some jobs. How much money does she have now?

Does not show
comprehension
of the problem.



The girl was happy because
she had money in the bank.
What does this show?



WORD PROBLEMS CONNECTIONS

“One of the first and foremost duties of the teacher is not to give his students the impression that mathematical problems have little connections with each other, and no connection at all with anything else... The teacher should encourage the students to imagine cases in which they could utilize again the procedure used, or apply the result obtained” Polya’s *How to Solve It*(p.15-16).

HOW STUDENTS LEARN?

- Piaget's work on Schema
- Piaget (1952) defined a schema as '*a cohesive, repeatable action sequence possessing component actions that are tightly interconnected and governed by a core meaning*.'
- Basic building block of intelligent behavior – a way of organizing knowledge
- “A set of linked mental representations of the world”

Source: <http://www.simplypsychology.org/piaget.html#schema>



THE PROCESS OF INTELLECTUAL GROWTH - ADAPTATION

○ Assimilation

- Using existing schema to deal with new situations
- Using the same procedures, a student solves a different type of problem.

○ Accommodation

- Existing schema (knowledge) does not work, the schema is changed to deal with a new situation
- A student changes the procedures to solve a new problem when the old procedures did not work.

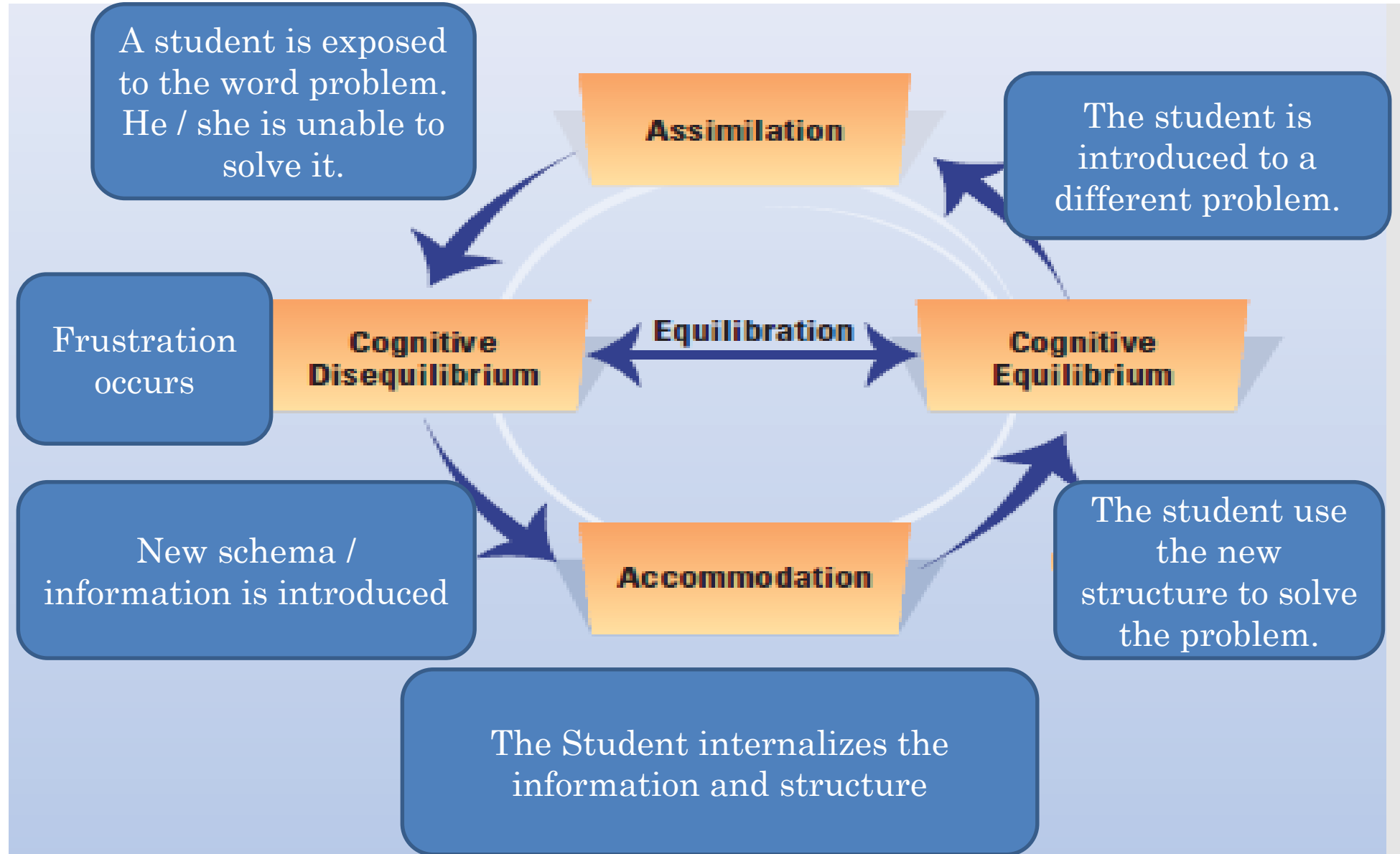
○ Equilibration

- The force to move from dis-equilibrium to equilibration accommodation

Source: : <http://www.simplypsychology.org/piaget.html#schema>



EXAMPLE OF A SITUATION



IMPLICATION FOR EDUCATION

- Piaget's constructivist and stage of development supports students using concrete models to represent math problems.
- Piaget's Schema model: “ Children who have severely limited interactions with their environments simply will not have the opportunities to develop and reorganize their cognitive structures so as to achieve mature ways of thinking.” (Cook)

Source: Cook, J. and G (2005) *Child Development Principles & Perspectives*, Pearson



WHAT NOW!

We may need to reconsider what to teach?

What are the word problem?



ADDITION/SUBTRACTION WORD PROBLEMS

Table 2: Addition and subtraction situations by grade level.			
	Result Unknown	Change Unknown	Start Unknown
Add To	<p><i>A</i> bunnies sat on the grass. <i>B</i> more bunnies hopped there. How many bunnies are on the grass now?</p> $A + B = \square$	<p><i>A</i> bunnies were sitting on the grass. Some more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies hopped over to the first <i>A</i> bunnies?</p> $A + \square = C$	<p>Some bunnies were sitting on the grass. <i>B</i> more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies were on the grass before?</p> $\square + B = C$
Take From	<p><i>C</i> apples were on the table. I ate <i>B</i> apples. How many apples are on the table now?</p> $C - B = \square$	<p><i>C</i> apples were on the table. I ate some apples. Then there were <i>A</i> apples. How many apples did I eat?</p> $C - \square = A$	<p>Some apples were on the table. I ate <i>B</i> apples. Then there were <i>A</i> apples. How many apples were on the table before?</p> $\square - B = A$
	Total Unknown	Both Addends Unknown ¹	Addend Unknown ²
Put Together /Take Apart	<p><i>A</i> red apples and <i>B</i> green apples are on the table. How many apples are on the table?</p> $A + B = \square$	<p>Grandma has <i>C</i> flowers. How many can she put in her red vase and how many in her blue vase?</p> $C = \square + \square$	<p><i>C</i> apples are on the table. <i>A</i> are red and the rest are green. How many apples are green?</p> $A + \square = C$ $C - A = \square$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	<p>"How many more?" version. Lucy has <i>A</i> apples. Julie has <i>C</i> apples. How many more apples does Julie have than Lucy?</p> <p>"How many fewer?" version. Lucy has <i>A</i> apples. Julie has <i>C</i> apples. How many fewer apples does Lucy have than Julie?</p> $A + \square = C$ $C - A = \square$	<p>"More" version suggests operation. Julie has <i>B</i> more apples than Lucy. Lucy has <i>A</i> apples. How many apples does Julie have?</p> <p>"Fewer" version suggests wrong operation. Lucy has <i>B</i> fewer apples than Julie. Lucy has <i>A</i> apples. How many apples does Julie have?</p> $A + B = \square$	<p>"Fewer" version suggests operation. Lucy has <i>B</i> fewer apples than Julie. Julie has <i>C</i> apples. How many apples does Lucy have?</p> <p>"More" version suggests wrong operation. Julie has <i>B</i> more apples than Lucy. Julie has <i>C</i> apples. How many apples does Lucy have?</p> $C - B = \square$ $\square + B = C$

Source: <http://achievethecore.org/page/932/situation-types>

DIFFERENT SITUATIONS OF ADDITION/SUBTRACTION WORD PROBLEMS

Schema
Structure

Represents information
unknown

Change

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	<p><i>A</i> bunnies sat on the grass. <i>B</i> more bunnies hopped there. How many bunnies are on the grass now?</p> $A + B = \underline{\hspace{1cm}}$ <p style="text-align: right;">(k)</p>	<p><i>A</i> bunnies were sitting on the grass. Some more bunnies joined them. Then there were <i>C</i> bunnies. How many bunnies joined the <i>A</i> bunnies?</p> $A + \underline{\hspace{1cm}} = C$ <p style="text-align: right;">(1)</p>	<p>Some bunnies were sitting on the grass. <i>B</i> more bunnies joined them. Then there were <i>C</i> bunnies. How many bunnies were on the grass before?</p> $\underline{\hspace{1cm}} + B = C$ <p style="text-align: right;">(2)</p>
TAKE FROM	<p><i>C</i> apples were on the table. I ate <i>B</i> apples. How many apples are on the table now?</p> $C - B = \underline{\hspace{1cm}}$ <p style="text-align: right;">(k)</p>	<p><i>C</i> apples were on the table. I ate some apples. There were <i>A</i> apples left. How many apples did I eat?</p> $C - \underline{\hspace{1cm}} = A$ <p style="text-align: right;">(1)</p>	<p>Some apples were on the table. I ate <i>B</i> apples. There were <i>A</i> apples left. How many apples were on the table before?</p> $\underline{\hspace{1cm}} - B = A$ <p style="text-align: right;">(2)</p>

Combine

	TOTAL UNKNOWN	BOTH ADDENDS UNKNOWN	ADDEND UNKNOWN
PUT TOGETHER/ TAKE APART	<p><i>A</i> red apples and <i>B</i> green apples are on the table. How many apples are on the table?</p> $A + B = \underline{\hspace{1cm}}$ <p style="text-align: right;">(k)</p>	<p>Grandma has <i>C</i> flowers. How many can she put in her red vase and how many in her blue vase?</p> $C = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} \quad \text{OR} \quad \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = C$ <p style="text-align: right;">(k)</p>	<p><i>C</i> apples are on the table. <i>A</i> are red and the rest are green. How many apples are green?</p> $C - A = \underline{\hspace{1cm}} \quad \text{OR} \quad A + \underline{\hspace{1cm}} = C$ <p style="text-align: right;">(1)</p>

Compare /
Difference

	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
COMPARE	<p>"How many more?" version. Lucy has <i>A</i> apples. Jack has <i>C</i> apples. How many more apples does Jack have than Lucy?</p> $A + \underline{\hspace{1cm}} = C \quad \text{OR} \quad C - A = \underline{\hspace{1cm}}$ <p style="text-align: right;">(1)</p>	<p>"More" version suggests operation. Jack has <i>B</i> more apples than Lucy. Lucy has <i>A</i> apples. How many apples does Jack have?</p> $A + B = \underline{\hspace{1cm}}$ <p style="text-align: right;">(1)</p>	<p>"Fewer" version suggests operation. Lucy has <i>B</i> fewer apples than Jack. Jack has <i>C</i> apples. How many apples does Lucy have?</p> $C - B = \underline{\hspace{1cm}} \quad \text{OR} \quad \underline{\hspace{1cm}} + B = C$ <p style="text-align: right;">(1)</p>

SCHEMA STRUCTURES

○ CHANGE

- Start with an amount which increases or decreases.
- Has a beginning, change, and ending.
- Verbs are critical to determine the change (increase or decrease).

○ COMBINE

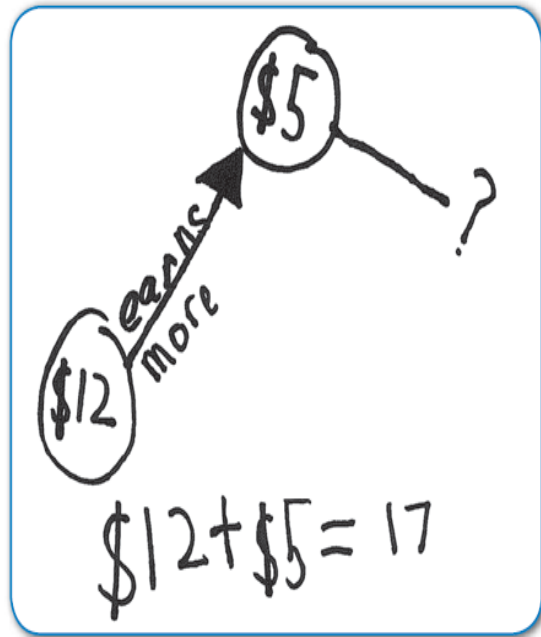
- Starts with groups that are put together for a total.
- Items in the groups are static.
- The word problem shares information about two groups.

○ COMPARE (DIFFERENCE)

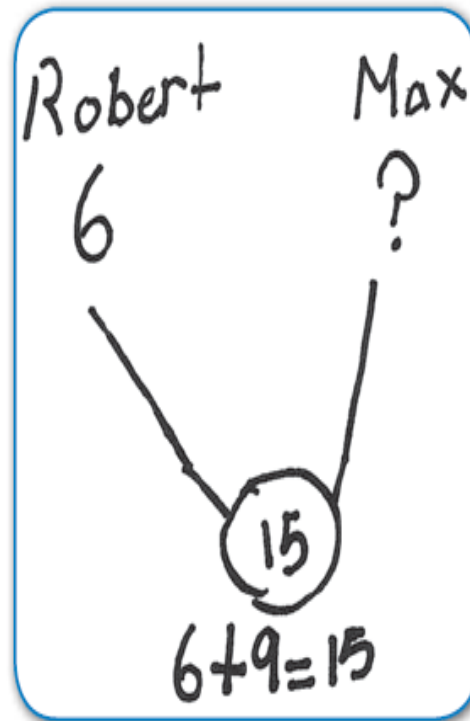
- Two amounts are compared to determine the difference.
- Items in the groups are static.
- The word problem has compare words (taller than, more than, less than....?).

SCHEMATIC DIAGRAM

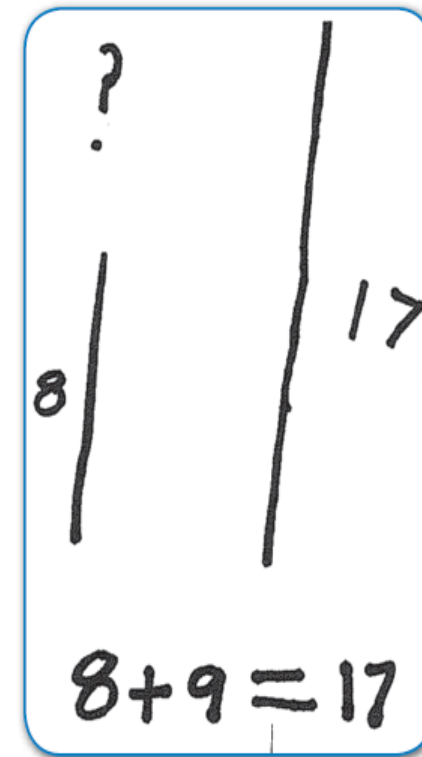
Schematic diagram is a visual tool to assist students in paring down information so that only the important structural information remains.



Change



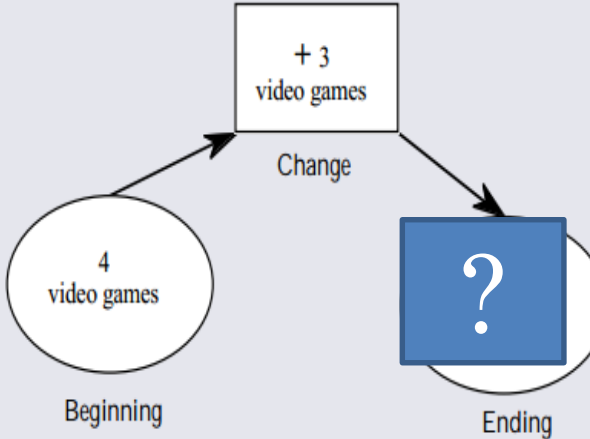
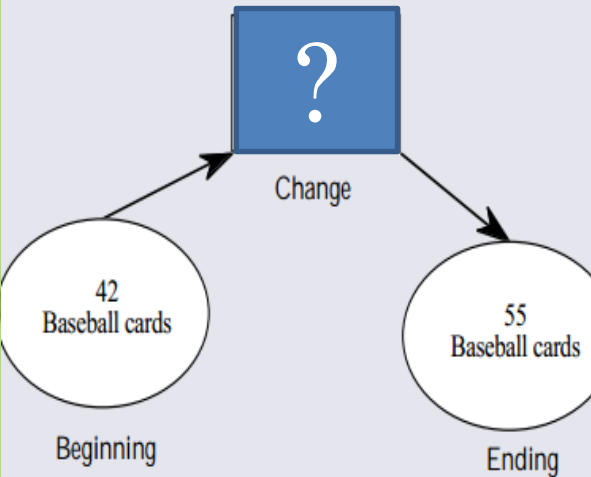
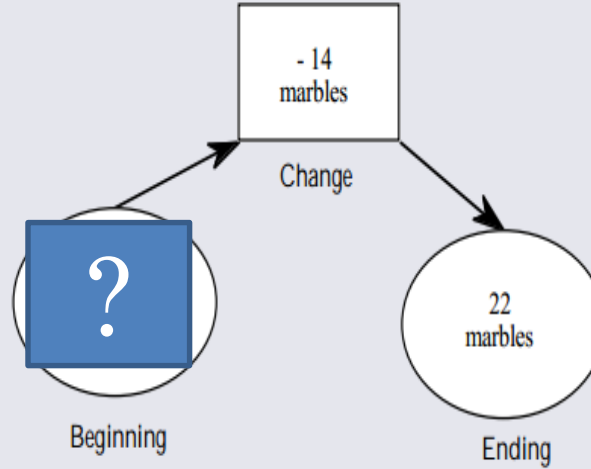
Combine



Compare



CHANGE Schema Structure

Result Unknown $4 + 3 = ?$	Change Unknown $42 + ? = 55$	Start Unknown $? - 14 = 22$
<p>Jane had 4 video games. Then her mother gave her 3 more video games for her birthday. How many video games does Jane have now?</p> 	<p>Tom had 42 baseball cards. He go to the store to buy more and now he has 55 baseball cards. How many cards did he buy?</p> 	<p>James has a marble collection. He gives away 14 marbles and now he has 22 marbles. How many marbles did James have before he gave some away?</p> 



SOLVE THIS CHANGE WORD PROBLEMS.

Draw the schema diagram on the whiteboard:

Roger had 36 comic books. Then his father bought him some more for his birthday. Roger now has 52 comic books. How many comic books did he receive from his father?

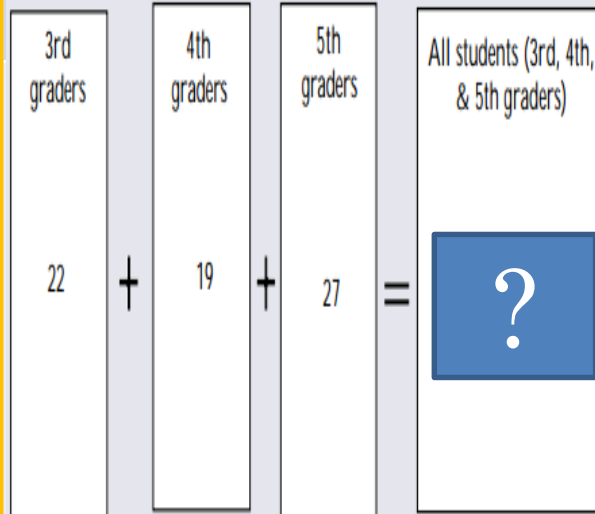


COMBINE Schema Structure

Total Unknown

$$22 + 19 + 27 = ?$$

Students at Hillcrest Elementary took part in the school play. There were 22 third graders, 19 fourth graders, and 27 fifth graders in the play. How many total students were in the play?

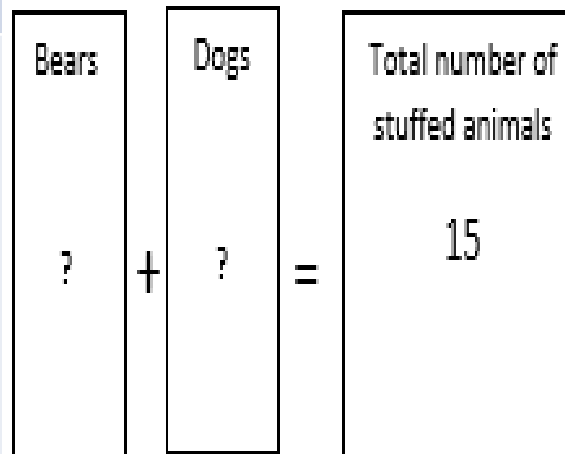


Both Addends

Unknown

$$? + ? = 15$$

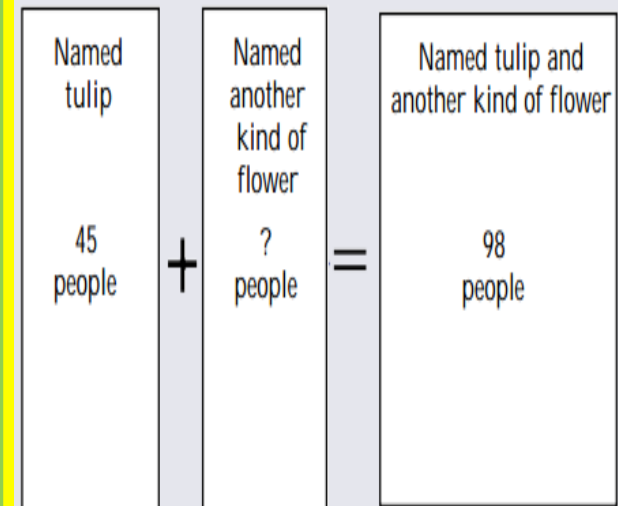
Eleanor has 15 stuffed animals. Some are bears and some are dogs. How many of each kind of stuffed animal could she have?



Addend Unknown

$$45 + ? = 98$$

In a survey, 98 people were asked what their favorite flower is and 45 named tulips. How many named another kind of flower?



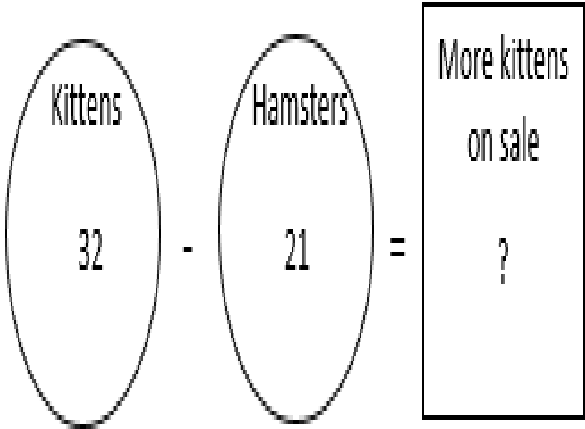
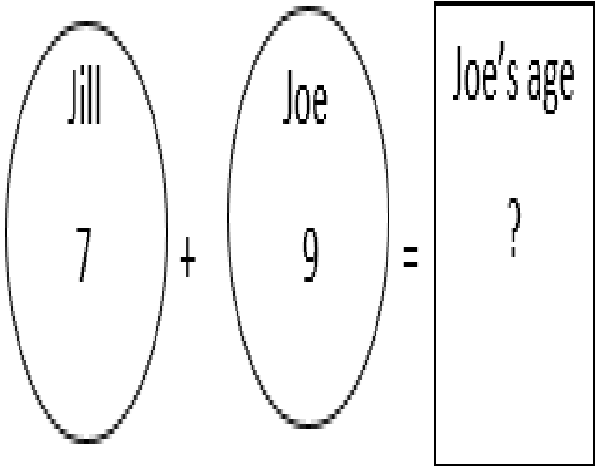
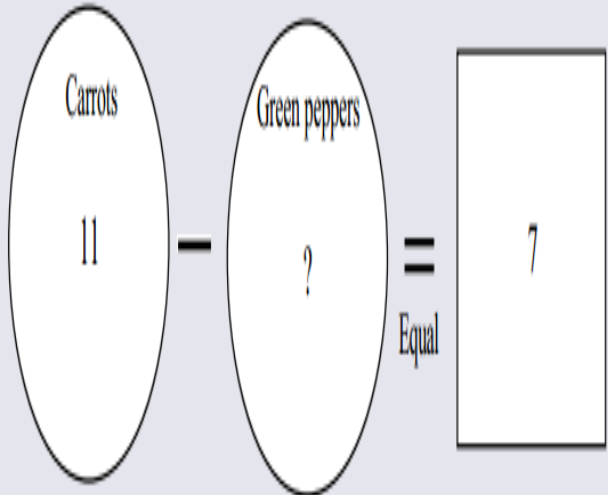
SOLVE THIS COMBINE WORD PROBLEMS.

Draw the schema diagram on the whiteboard:

In an apple picking contest, the third and fourth graders picked 84 apples. If the third graders picked 41 apples, how many apples did the fourth graders pick?



COMPARE Schema Structure

Difference Unknown $32 - 21 = ?$	Bigger Unknown $7 + 9 = ?$	Smaller Unknown $11 - ? = 7$
<p>The pet store is having a sale of 21 hamsters and 32 kittens. How many more kittens are on sale than the hamsters?</p>	<p>Jill is 7 years old. Joe is 9 years older than Jill. How old is Joe?</p>	<p>Steve picked 11 carrots. He picked 7 fewer green peppers than carrots. How many green peppers did Steve pick?</p>
		



SOLVE THIS COMPARE WORD PROBLEMS.

Draw the schema diagram on the whiteboard:

Janice is 85 centimeters tall. She is 16 centimeters taller than Melinda. How tall is Melinda?



SCHEMA-BASED INSTRUCTION VS. GENERAL STRATEGY INSTRUCTION

Schema-Based Instruction	General Strategy Instruction
<ul style="list-style-type: none">• Read to understand	<ul style="list-style-type: none">• Read to understand
<ul style="list-style-type: none">• Identify the problem type, and use the schema diagram to represent the problem	<ul style="list-style-type: none">• Draw a picture / visual to represent the problem
<ul style="list-style-type: none">• Transform the diagram to a math sentence, and solve the problem	<ul style="list-style-type: none">• Solve the problem
<ul style="list-style-type: none">• Look back to check	<ul style="list-style-type: none">• Look back to check

ACTIVITY

- Each participant will work independently on completing the following task:
 1. Determine the problem type: change, combine, compare
 2. Organize the information using the schematic diagram, including labels.
 3. Plan to solve the problem: (i.e., $3 + 4 = ?$).
 4. Solve the problem: (i.e., $3 + 4 = 7$ friends).



SCHEMATIC BASED INSTRUCTION

- A Schema is a framework, outline or plan for solving a problem
- Schematic Based Instruction (SBI) is explicit and structured
- Students are taught
 - To recognize problems as falling within word-problem types
 - To apply a solution method that matches that problem type

Jitendra,A. *Meeting the needs of students with learning disabilities: the role of schema-based instruction.* NCTM 2008 Annual Convention



PHASE 1 OF SCHEMA TRAINING

➤ Change Schematic Diagram:

- Isolate key information
- Draw the schema comprehension drawing first - “Picture that shows only the really important information”
- Start with no unknown amount
- Gradually, introduce Result Unknown, then Change Unknown and finally Start Unknown— (when students are successful with correctly identifying the schemas)
- Time Frame: 3 weeks 2 days a week

PHASE 2 OF SCHEMA TRAINING

➤ Compare Schematic Diagram

- Continue to work with change schematic diagram
- First Teach students stories that seek compare two amounts – focus on compare words as signal words
- Started with no unknown amount
- Time Frame: 4 weeks 2 days a week



PHASE 3 OF SCHEMA TRAINING

➤ **Combine Schematic Diagram**

- Continue to work with change and compare schematic diagram
- Draw the schema comprehension drawing first of a stories that link two static groups together
- Started with no unknown amount
- Time Frame: 4 weeks 2 days a week

Addition / Subtraction Word Problems

Equation

Comprehension

Structure
(Schema)

